Изследване на функции

За това домашно, по познатия вече начин, ползвайте следната формула

$$\mathbf{1} + \text{остатъка при деление на } \boldsymbol{k} \text{ на сумата } (\boldsymbol{mn} + \mathbf{12}\boldsymbol{z})$$

Модул А

Задача 1. ($\boldsymbol{k} = \mathbf{8}$) За дадените функции намерете:

	А: първия диференциал		Б: третия диференциал
1.1	$f_1(x) = 2x - x^3\sin(x) + \frac{2^x}{x^2}$; $f_2(y) = 2x.\operatorname{tg}(y) - y^3\arcsin(x) + \frac{2^x}{y^2}$; $f_3(x) = \arcsin(\sqrt{2x^2 - 1})$;	1.1	$f_1(x) = \frac{1}{6}x^4 - \sin\left(2x - \frac{\pi}{6}\right)$; $f_2(x) = x.\sin(x)$;
1.2	$f_1(x) = \ln(x) - x^3\arcsin(x) + \frac{3^x}{x^3}$; $f_2(y) = \ln(x).\operatorname{cotg}(y) - y^x + \frac{y^x}{x^3}$; $f_3(x) = \arccos(\sqrt{x^2 + 1})$	1.2	$f_1(x) = x - \sin\left(\frac{\pi}{6} - 3x\right)$; $f_2(x) = e^{-x}.\cos(x)$;
1.3	$f_1(x) = 7\cos(x) - x^3\operatorname{arctg}(x) + \frac{e^x}{x^e}$; $f_2(t) = \ln(t)\operatorname{cotg}(x) - t^x + \frac{x^t}{2^x}$; $f_3(x) = \operatorname{arccotg}(\sqrt{\sin(x) - 1})$;	1.3	$f_1(x) = x^3 + \cos\left(2x - \frac{\pi}{3}\right)$; $f_2(x) = x^2.\sin(x)$;
1.4.	$f_1(x) = \operatorname{arctg}(x) - e^x\sin(x) + \frac{x^2}{2^x}$; $f_2(y) = 2x.\operatorname{cotg}(y) - y^3\arccos(x) + \frac{3^y}{x^2}$; $f_3(x) = \arccos(\sqrt{x^2 - 2^x})$;	1.4	$f_1(x) = \frac{1}{6}x^6 - \sin(2x - \pi)$; $f_2(x) = e^{-x}.\sin(x)$;
1.5.	$f_1(x) = \operatorname{arccotg}(x) - x^3\ln(x) + \frac{x^5}{5^x}$; $f_2(y) = x.\operatorname{arctg}(y) - y^3\ln(2x) + \frac{y}{x^2}$; $f_3(x) = \operatorname{arctg}(\sqrt{x^2 - 2x - 1})$;	1.5	$f_1(x) = 2x^4 - \cos(3 - 2x)$; $f_2(x) = e^{2x}.\cos(x)$;
1.6	$f_1(x) = \arcsin(x) - x^2\operatorname{cotg}(x) + \frac{2^x}{x^4}$;	1.6	$f_1(x) = 3x^{-1} - \cos(2 - 3x)$;

$f_2(y) = 2^x \text{tg}(y) - x^3 \arcsin(y) + \frac{y^x}{x^2};$

$f_3(x) = \sqrt{2\ln(x^2-1)};$

1.7 $f_1(x) = \arcsin(x) - x^3 \sin(x) + \frac{x^3}{3^x};$

$f_2(y) = y^x \text{tg}(x) - y^3 \arcsin(2^x) + \frac{x^y}{x^2};$

$f_3(x) = \sin(\ln(2x-1));$

1.8 $f_1(x) = \arcsin(x) - x^3 \sin(x) + \frac{x^3}{3^x};$

$f_2(y) = \frac{\text{tg}(y)}{\sin(x)} - \sqrt{y^3} \arcsin(x) + \frac{e^x}{y^2};$

$f_3(x) = \arcsin^3(\sqrt{x^2-1});$

$f_2(x) = e^x. \cos(2x);$

1.7 $f_1(x) = 2e^{4x} - \sin(3\pi - x);$

$f_2(x) = e^{-2x}. \cos(x);$

1.8 $f_1(x) = 2e^{-x} - \cos(\pi - x);$

$f_2(x) = e^x. \cos(3x);$

Напишете уравнението на допирателната към графиката на функцията $y = f(x)$ в точката $M_0(x_0; f(x_0))$, ако:

1.1) $y = x^3 - 3x^2 - 9x + 1,\ x_0 = 1;$

1.2) $y = x^3 - 6x^2 + 9x + 1,\ x_0 = -1;$

1.3) $y = x^3 + 3x^2 - 45x + 1,\ x_0 = 0;$

1.4) $y = 2x^3 - 21x^2 + 60x - 15, x_0 = 2;$

1.5) $y = 2x^3 - 6x^2 - 18x - 5,\ x_0 = -1;$

1.6) $y = 2x^3 - 9x^2 - 24x + 5,\ x_0 = 1;$

1.7) $y = x^3 - 9x^2 + 15x + 5,\ x_0 = 2;$

1.8) $y = x^3 - 3x^2 - 9x + 1,\ x_0 = 0;$

Задача 2. (***k* = 5**) Намерете границите:

2.1) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2+x-6};$ $\lim_{x\to 2} \frac{\sin(x-2)}{3x-6};$ $\lim_{x\to 0} \frac{e^x - e^{-x}}{sin(2x)};$ $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right);$

2.2) $\lim_{x\to 2} \frac{x^2-7x+10}{x^2+x-6};$ $\lim_{x\to \pi} \frac{5(x-\pi)}{\sin(3x-3\pi)};$ $\lim_{x\to 0} \frac{e^{2x} - e^{-2x}}{sin(x)};$ $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right);$

2.3) $\lim_{x\to 2} \frac{x^2+5x-14}{x^2+x-6};$ $\lim_{x\to -2} \frac{\sin(x+2)}{2x+4};$ $\lim_{x\to \frac{\pi}{2}} \frac{e^x - e^{-x}}{cos(x)};$ $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x-1}\right),$

2.4) $\lim_{x\to 2} \frac{x^2-7x+10}{x^2+3x-10};$ $\lim_{x\to 2} \frac{\sin(4x-8)}{x-2};$ $\lim_{x\to 0} \frac{e^{-x} - e^{x}}{sin(3x)};$ $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin(\pi-x)}\right);$

2.5) $\lim_{x\to -2} \frac{x^2+7x+10}{x^2-3x-10};$ $\lim_{x\to \pi} \frac{\sin(x-\pi)}{\sin(3x-3\pi)};$ $\lim_{x\to 0} \frac{e^{-3x} - e^{3x}}{sin(5x)};$ $\lim_{x\to 2} \left(\frac{x}{x-2} - \frac{1}{\ln(x-1)}\right);$

Задача 3. ($\boldsymbol{k} = 7$) Определете интервалите на монотонност на функциите:

3.1) $y = x^3 - 3x^2 - 9x + 1$; $y = x(1 - \sqrt{x}), x \in (0; +\infty)$;

3.2) $y = x^3 - 6x^2 + 9x + 1$; $y = \frac{x}{2} - \sin(x), x \in [0; 2\pi]$;

3.3) $y = x^3 + 3x^2 - 45x + 1$; $y = \ln(1 - x^2), x \in (-1; 1)$;

3.4) $y = 2x^3 - 21x^2 + 60x - 15$; $y = x(3 - \sqrt{x}), x \in (0; +\infty)$;

3.5) $y = 2x^3 - 6x^2 - 18x - 5$; $y = x - 2\sin(\pi - x), x \in [0; 2\pi]$;

3.6) $y = 2x^3 - 9x^2 - 24x + 5$; $y = \ln(4 - x^2), x \in (-2; 2)$;

3.7) $y = 2x^3 - 18x^2 + 30x + 5$; $y = x - \ln(2 - x^2), x \in (-\sqrt{2}; \sqrt{2})$.

Задача 4. ($\boldsymbol{k} = 6$) Определете локалните екстремуми на функциите:

4.1) $y = x^3 - 6x^2 + 9x + 5$; $y = x - 2\sin(\pi - x), x \in [0; 2\pi]$;

4.2) $y = 2x^3 - 21x^2 + 60x - 15$; $y = x(1 + \sqrt{x}), x \in (0; +\infty)$;

4.3) $y = 2x^3 - 6x^2 - 18x - 5$; $y = \frac{x}{2} - \sin(x), x \in [0; 2\pi]$;

4.4) $y = 2x^3 - 9x^2 - 24x + 5$; $y = \ln(1 - x^2), x \in (-1; 1)$;

4.5) $y = x^3 + 3x^2 - 45x + 1$; $y = \ln(4 - x^2), x \in (-2; 2)$;

4.6) $y = x^3 - 3x^2 - 9x + 1$; $y = x(3 - \sqrt{x}), x \in (0; +\infty)$.

Задача 5. ($\boldsymbol{k} = 5$) Намерете абсолютните екстремуми на функциите:

5.1) $y = 2x^3 - 9x^2 - 24x + 5$; $x \in [-7; 3]$; $y = \ln(4 - x^2), x \in [-1; 1]$;

5.2) $= 2x^3 - 6x^2 - 18x - 5$; $x \in [-2; 4]$; $y = x(3 - \sqrt{x}), x \in [1; 4]$

5.3) $y = x^3 + 3x^2 - 45x + 1, x \in [-7; 4]$; $y = \ln(1 - x^2), x \in \left[-\frac{1}{2}; \frac{1}{2}\right]$;

5.4) $y = x^3 - 3x^2 - 9x + 1$, $x \in [-1; 4]$; $y = x(1 + \sqrt{x}), x \in [1; 5]$;

5.5) $y = x^3 - 6x^2 + 9x + 5$, $x \in [-2; 5]$; $y = \frac{x}{2} - \sin(x), x \in [-\pi; \pi]$.

Задача 6. ($\boldsymbol{k} = 9$) Определете интервалите на изпъкналост, вдлъбнатост и инфлексните точки на функциите:

6.1) $y = x^3 - 3x^2 - 9x + 1$; $y = x^2 - x(1 - \sqrt{x}), x \in (0; +\infty)$;

6.2) $y = x^3 - 6x^2 + 9x + 1$; $y = \frac{x^2}{2} - \sin(x), x \in [0; 2\pi]$;

6.3) $y = x^3 + 3x^2 - 45x + 1$; $y = \ln(1 - x^2)\,, x \in (-1; 1)$;

6.4) $y = 2x^3 - 21x^2 + 60x - 15$; $y = 3x^2 + 3x - x\sqrt{x} + 5, x \in (0; +\infty)$;

6.5) $y = 2x^3 - 6x^2 - 18x - 5$; $y = x - 2\sin(\pi - x)\,, x \in [0; 2\pi]$;

6.6) $y = 2x^3 - 9x^2 - 24x + 5$; $y = x\left(1 + \sqrt{x}\right), x \in (0; +\infty)$;

6.7) $y = 2x^3 - 18x^2 + 30x + 5$; $y = x - \ln(2 - x)\,, x \in (-\infty; 2)$;

6.8) $y = x^3 - 3x^2 - 9x + 1$; $y = x - \ln(4 - x^2)\,, x \in (-2; 2)$;

6.9) $y = x^3 - 6x^2 + 9x + 1$; $y = \frac{x}{2} - \sin(2x)\,, x \in [0; \pi]$.

Задача 7. (***k* = 3**) Намерете асимптотите към графиките на функциите:

7.1) $y = \frac{x^2 - 6x + 3}{x - 3}$; 7.2) $y = \frac{1}{(x-3)^2}$; 7.3) $y = \frac{x^3}{x^2 + 9}$.

Модул Б

Задача 8. (***k* = 1**) Да се направи пълно изследване на функцията $y = x^2 e^{-x}$.

Пожелавам ви приятно и успешно решаване на тези задачи!

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