Неопределен интеграл

За това домашно, по познатия вече начин, ползвайте следната формула

$$1 + \text{остатъка при деление на } \boldsymbol{k} \text{ на сумата } (\boldsymbol{2mn + 13z})$$

Модул А

Задача 1. ($k = 8$) Решете следните интеграли, чрез непосредствено интегриране:

1.1. **a)** $\int \left(3x^2 - 2x + \frac{3}{x} + 5\sqrt[3]{x^2}\right)dx;$ $\int \frac{5-2x+3x^3}{x}dx;$ $\int \frac{2}{x^2+9}dx;$ $\int \frac{2}{\sqrt{6-x^2}}dx;$

  б) $\int \sin(2x - 3)\,dx;$ $\int \sin^2(x - 3)\,dx;$ $\int \frac{2}{\sin^2(2x-3y)}dy$

1.2. **a)** $\int \left(4x^3 - x + \frac{5}{x} + 8\sqrt[5]{x^3}\right)dx;$ $\int \frac{4x^5-2x+3x^3}{x^2}dx;$ $\int \frac{2}{x^2-9}dx;$ $\int \frac{2}{\sqrt{6+x^2}}dx;$

  б) $\int \cos(3x - 2)\,dx;$ $\int \cos^2(x + 3)\,dx;$ $\int \frac{2}{\cos^2(2ux-3)}du$

1.3. **a)** $\int \left(12x^2 - x + \frac{2}{x} + 7\sqrt[4]{x^3}\right)dx;$ $\int \frac{6x^5-2x+3x^2}{x^3}dx;$ $\int \frac{2}{4-x^2}dx;$ $\int \frac{2}{\sqrt{3-x^2}}dx;$

  б) $\int 3^{3x-5}dx;$ $\int \cos^2\left(\frac{x-3}{2}\right)dy;$ $\int \frac{2}{\sin^2(3x-2u)}du$

1.4. **a)** $\int \left(6x^2 - 3^x + \frac{2}{x} + 4\sqrt[3]{x}\right)dx;$ $\int \frac{6x^5+3x+x^2}{x^3}dx;$ $\int \frac{2}{7-x^2}dx;$ $\int \frac{2}{\sqrt{x^2-5}}dx;$

  б) $\int e^{3x-2}dx;$ $\int \sin^2\left(\frac{2x-3}{2}\right)dy;$ $\int \frac{2}{\sin^2(3xu-x)}du$

1.5. **a)** $\int \left(30x^5 - 2x + \frac{3}{x} + 5\sqrt[3]{x^2}\right)dx;$ $\int \frac{5-2x+3x^3}{x^2}dx;$ $\int \frac{2}{x^2-9}dx;$ $\int \frac{2}{\sqrt{x^2-3}}dx;$

  б) $\int \frac{2}{3x-5}dx;$ $\int \sin^2\left(\frac{2-3x}{2}\right)dx;$ $\int \frac{2}{\cos^2(2x-3y)}dy$

1.6. **a)** $\int \left(30x^4 - 12x - \frac{5}{x} + 9\sqrt[7]{x^2}\right)dx;$ $\int \frac{5-2x+3x^3}{x^4}dx;$ $\int \frac{2}{x^2-3}dx;$ $\int \frac{3}{\sqrt{x^2-1}}dx;$

  б) $\int \frac{3}{5-3x}dx;$ $\int 2^{2x-y}\,dy;$ $\int \frac{2}{\cos^2(2x-3)}dx$

1.7. **a)** $\int \left(30x^4 - 12e^x - \frac{5}{x^2} + 8\sqrt[5]{x^3}\right)dx;$ $\int \frac{3-x+x^3}{x^2}dx;$ $\int \frac{2}{x^2+3}dx;$ $\int \frac{3}{\sqrt{x^2+1}}dx;$

  б) $\int \frac{3}{7-3x}dx;$ $\int e^{2x-3y}\,dy;$ $\int \frac{2}{\sin^2(2-3x)}dx$

1.8. **a)** $\int \left(15x^4 - 2^x - \frac{3}{x^4} + 13\sqrt[8]{x^5}\right)dx;$ $\int \frac{4x-3+x^3}{x^4}dx;$ $\int \frac{2}{x^2+1}dx;$ $\int \frac{3}{\sqrt{x^2-2}}dx;$

  б) $\int \frac{3}{5-x}dx;$ $\int e^{-x}\,dx;$ $\int \frac{2}{\sin^2(2y-3x)}dy$

Задача 2. ($k = 9$) Решете следните интеграли, чрез внасяне на израз зад знака на диференциала:

2.1. 1) $\int \frac{8x}{\sqrt{5-x^2}}dx;$ 2) $\int \frac{\cos(x)}{\sin^2(x)-9}dx;$ 3) $\int \text{cotg}(2x)dx;$

  4) $\int e^x \sin(6e^x - 5)\,dx;$ 5) $\int \frac{2 - \sqrt[4]{\ln^3(x)}}{x}dx$

2.2. 1) $\int \frac{2x}{x^2+1}dx;$ 2) $\int \frac{\sin(x)}{\cos^2(x)-2}dx;$ 3) $\int \text{tg}(2 - x)dx;$

  4) $\int e^{-x} \sin(e^{-x})\,dx;$ 5) $\int \frac{4\ln^3(x) - 1}{x}dx$

2.3. 1) $\int \frac{2x^2}{x^3+1}dx;$ 2) $\int \frac{\sin(x)}{\cos^3(x)}dx;$ 3) $\int 6\text{tg}(x).\cos^3(x)dx;$

4) $\int 2^x \sin(2^x)\,dx$; 5) $\int \dfrac{3\ln^2(x)-2}{x}\,dx$

2.4. 1) $\int \dfrac{x^2}{\sqrt[3]{x^3+1}}\,dx$; 2) $\int \cos^3\left(\dfrac{x}{2}\right)dx$; 3) $\int \dfrac{6\operatorname{tg}(x)}{\cos^2(x)}\,dx$;

4) $\int e^{2x}\sin(e^{2x})\,dx$; 5) $\int \dfrac{2x-5\ln^4(x)}{x}\,dx$

2.5. 1) $\int \dfrac{x^3}{\sqrt[3]{x^4+1}}\,dx$; 2) $\int \sin^3\left(\dfrac{x}{2}\right)dx$; 3) $\int \dfrac{6\operatorname{cotg}^2(x)}{\sin^2(x)}\,dx$;

4) $\int e^{2x}\sin(e^{2x})\,dx$; 5) $\int \dfrac{x-3\ln^2(x)}{x}\,dx$

2.6. 1) $\int 8x^3\sin(3x^4)\,dx$; 2) $\int \sin^3\left(\dfrac{x}{4}\right)dx$; 3) $\int 6\operatorname{cotg}(3x)\,dx$;

4) $\int 2^x\cdot\sqrt[3]{2^x-3}\,dx$; 5) $\int \dfrac{10\sqrt[7]{\ln^3(x)}-1}{x}\,dx$

2.7. 1) $\int \dfrac{2x^2}{x^3+1}\,dx$; 2) $\int \dfrac{2\sin(2x)}{\cos^5(2x)}\,dx$; 3) $\int \operatorname{tg}(x).\cos^{-2}(x)\,dx$;

4) $\int 3^x\sin(3^x)\,dx$; 5) $\int \dfrac{\sqrt{\ln(x)-2}}{x}\,dx$

2.8. 1) $\int \dfrac{3x^2}{x^6+1}\,dx$; 2) $\int \dfrac{\operatorname{tg}(x)}{\cos^3(x)}\,dx$; 3) $\int 8\operatorname{tg}^3(x).\cos^{-2}(x)\,dx$;

4) $\int e^{-x}\sin(e^{-x})\,dx$; 5) $\int \dfrac{3ln^2(x)-2x}{x}\,dx$

2.9. 1) $\int \dfrac{5x^4}{x^{10}+1}\,dx$; 2) $\int \dfrac{\operatorname{cotg}(x)}{\sin^3(x)}\,dx$; 3) $\int \operatorname{cotg}(x).\sin^{-2}(x)\,dx$;

4) $\int \dfrac{2^x}{cos^2(2^x)}\,dx$; 5) $\int \dfrac{5ln^4(x)-2\sqrt{x}}{x}\,dx$

Задача 3. ($k=5$) Решете интегралите, чрез интегриране по части:

3.1. 1) $\int x.\sin(x)\,dx$; 2) $\int x.e^{-2x}\,dx$; 3) $\int 4x.\ln(x)\,dx$; 4) $\int x^2\sin(6x-5)\,dx$.

3.2. 1) $\int (3-x).\cos(x)\,dx$; 2) $\int x.2^{-x}\,dx$; 3) $\int 4x^3.\ln(x)\,dxdx$; 4) $\int (4x)^3\ln^2(x)\,dx$.

3.3. 1) $\int (2x-1).2^x\,dx$; 2) $\int x.\sin(3x-7)\,dx$; 3) $\int 10x^4.\log_2(x)\,dxdx$; 4) $\int x^2\sin(x-5)\,dx$.

3.4. 1) $\int (x+2).e^x\,dx$; 2) $\int x.\cos(2x+1)\,dx$; 3) $\int \operatorname{arctg}(x)\,dx$; 4) $\int (3x)^2\ln^2(x)\,dx$.

3.5. 1) $\int (x-1).\cos(x)\,dx$; 2) $\int x.\cos\left(\dfrac{x+1}{2}\right)dx$; 3) $\int \arcsin(x)\,dx$; 4) $\int x^2\cos(2x-1)\,dx$.

Задача 4. ($k=7$) Решете интегралите, чрез полагане:

4.1. 1) $\int \dfrac{2}{\sqrt{3-2x-x^2}}\,dx$; 2) $\int \dfrac{1}{\sqrt{x+2}-\sqrt[3]{x+2}}\,dx$; 3) $\int \dfrac{1}{2+\sin(x)}\,dx$; 4) $\int \dfrac{6x-2}{\sqrt{3x-1}}\,dx$.

4.2. 1) $\int \dfrac{1}{\sqrt{3+2x-x^2}}\,dx$; 2) $\int \dfrac{1}{\sqrt{x-1}-\sqrt[3]{x-1}}\,dx$; 3) $\int \dfrac{1}{2-\sin(x)}\,dx$; 4) $\int \dfrac{4x-3}{\sqrt{2x-3}}\,dx$.

4.3. 1) $\int \dfrac{1}{\sqrt{x^2+6x-10}}\,dx$; 2) $\int \dfrac{12x-5}{\sqrt{2-3x}}\,dx$; 3) $\int \dfrac{1}{2+\cos(x)}\,dx$; 4) $\int \dfrac{2}{\sqrt{2x-1}-\sqrt[3]{2x-1}}\,dx$.

4.4. 1) $\int \dfrac{2}{\sqrt{x^2+2x-8}}\,dx$; 2) $\int \dfrac{1}{\sqrt{2-x}-\sqrt[3]{2-x}}\,dx$; 3) $\int \dfrac{1}{1+\cos(x)}\,dx$; 4) $\int \dfrac{1-x}{\sqrt{2x-3}}\,dx$.

4.5. 1) $\int \dfrac{1}{\sqrt{x^2+4x-5}}\,dx$; 2) $\int \dfrac{7-12x}{\sqrt{2x-3}}\,dx$; 3) $\int \dfrac{1}{2+\sin(x)}\,dx$; 4) $\int \dfrac{1}{\sqrt{x-2}-\sqrt[3]{x-2}}\,dx$.

4.6. 1) $\int \frac{5}{\sqrt{7-4x-x^2}}dx$;   2) $\int \frac{7-12x}{\sqrt{4x-3}}dx$;   3) $\int \frac{1}{1+\sin(x)}dx$;   4) $\int \frac{1}{\sqrt{4-x}-\sqrt[3]{4-x}}dx$.

Модул Б

Задача 5. ($k = 4$) Решете интегралите:

5.1. 1) $\int e^{2x}\sin(3x-1)dx$;   2) $\int \operatorname{cotg}^3(3x)dx$;   3) $\int \frac{4x^2-3x-7}{(x-3)(x^2+1)}dx$.

5.2. 1) $\int e^{-2x}\sin(2-x)dx$;   2) $\int \operatorname{tg}^2(2x)dx$;   3) $\int \frac{4x^2+9x+1}{(x+3)(x^2-1)}dx$.

5.3. 1) $\int e^{-3x}\sin(2-6x)dx$;   2) $\int \operatorname{cotg}^2(3x)dx$;   3) $\int \frac{4x^2+x+5}{(x-1)(x^2+4)}dx$.

5.4. 1) $\int e^{-x}\cos(2x-1)dx$;   2) $\int \operatorname{tg}^3(2x)dx$;   3) $\int \frac{6x^2+5x+39}{(x+1)(x^2+39)}dx$.

Задача 6. ($k = 5$) Намерете общото решение на дадените ОДУ.

6.1. 1) $y' = \sin(x) + 2x$;   2) $y'' = \sin(x) + 2x$;   3) $y''' = \sin(x)$.

6.2. 1) $y' = 3\sin(x) + 2$;   2) $y'' = e^x x^2 + 2x$;   3) $y''' = 2 - 48x$.

6.3. 1) $y' = 8\sin(2x) + 2x$;   2) $y'' = 12e^{2x} + \sin(x)$;   3) $y''' = \cos(x) + 2x$.

6.4. 1) $y' = \operatorname{tg}(x) + 2e^{-x}$;   2) $y'' = 3\cos(x) + 12xe^x$;   3) $y' = \sin(x) + 2x$.

6.5. 1) $y' = \operatorname{cotg}(x) + 2xe^{-x}$;   2) $y'' = (3x-1)\cos(x)$;   3) $y' = 27e^{3x} + 6$.

Задача 7. ($k = 3$) Намерете частно решение на дадените ОДУ, удовлетворяващо посочените допълнителни условия.

7.1. 1) $y' = 3x - \cos(2x)$ при $y(0) = 3$;   2) $y' = 3x - \cos(2x)$ при $y(2\pi) = 6\pi^2 - 3$;

   3) $x'' = \sin(t) + 6t, x(0) = -1, x'(0) = 4$.

7.2. 1) $y' = \sin(x) + 2x$ и $y(0) = 1$;   2) $y' = \sin(\pi x) - 1$ и $y(1) = 2\pi$;

   3) $x'' = \sin(t) + 2t, x(0) = 3, x'(0) = 2$.

7.3. 1) $y' = 3\sin(x) + 2$ и $y(\pi) = 2\pi - 3$;   2) $y' = 3\cos(x) + 2$ и $y(\pi) = 2\pi - 3$;

   3) $x'' = 3\cos(t) + 12t, x(0) = 1, x'(0) = 6$.

Допълнение - задачи за самоподготовка

Във всеки ред на таблицата има по 3 еднотипни задачи.

Непосредствено интегриране

Зад.	$j = 1$	Зад.	$j = 2$	Зад	$j = 3$
1.1	$\int \left(3x^2 - 2x + \frac{3}{x}\right)dx$	1.2	$\int \left(4x^3 - x + \frac{5}{x}\right)dx$	1.3	$\int \left(12x^2 - x + \frac{2}{x}\right)dx$
2.1	$\int \frac{5 - 2x + 3x^3}{x}dx$	2.2	$\int \frac{4x^5 - 2x + 3x^3}{x^2}dx$	2.3	$\int \frac{6x^5 - 2x + 3x^2}{x^3}dx$

3.1	$\int \dfrac{4x^5 - 5x + 3x^3}{x^7} dx$	3.2	$\int \dfrac{6x^5 - 8x + 3x^2}{x^6} dx$	3.3	$\int \dfrac{5 - 3x + 3x^3}{x^5} dx$
4.1	$\int (5a^2 x + 4a^3 x^3)\, da$	4.2	$\int (4x^5 y - 4x^3 y^3)\, dy$	4.3	$\int (4ay^5 - 5a^2 y)\, dy$
5.1	$\int \dfrac{4\sqrt[3]{x^5} - 5x\sqrt{x}}{x} dx$	5.2	$\int \dfrac{4x\sqrt[4]{x^3} - 5x\sqrt{x}}{\sqrt{x}} dx$	5.3	$\int \dfrac{4x^2\sqrt[3]{x^2} - 5x\sqrt{x}}{x} dx$
6.1	$\int \sin(2x - 3)\, dx$	6.2	$\int \cos(2x - 3)\, dx$	6.3	$\int \sin(3 - x)\, dx$
7.1	$\int sin^2(x - 3)\, dx$	7.2	$\int sin^2(3x - 1)\, dx$	7.3	$\int sin^2(2x - 3)\, dx$
8.1	$\int \dfrac{2}{sin^2(2x - 3)} dx$	8.2	$\int \dfrac{3}{sin^2(3x - 1)} dx$	8.3	$\int \dfrac{1}{sin^2(x - 3)} dx$
9.1	$\int \dfrac{2}{x^2 - 4} dx$	9.2	$\int \dfrac{2}{x^2 - 9} dx$	9.3	$\int \dfrac{2}{x^2 - 1} dx$
10.1	$\int \dfrac{2}{x^2 + 9} dx$	10.2	$\int \dfrac{2}{x^2 + 1} dx$	10.3	$\int \dfrac{2}{x^2 + 4} dx$
11.1	$\int \dfrac{3}{\sqrt{x^2 - 7}} dx$	11.2	$\int \dfrac{2}{\sqrt{x^2 - 3}} dx$	11.3	$\int \dfrac{1}{\sqrt{x^2 + 5}} dx$
12.1	$\int \dfrac{2}{\sqrt{6 - x^2}} dx$	12.2	$\int \dfrac{1}{\sqrt{3 - x^2}} dx$	12.3	$\int \dfrac{8}{\sqrt{5 - x^2}} dx$
13.1	$\int \dfrac{2x^2}{x^2 + 6} dx$	13.2	$\int \dfrac{3x^2}{x^2 + 3} dx$	13.3	$\int \dfrac{2x^2}{x^2 + 7} dx$
14.1	$\int tg^2(2x)dx$	14.2	$\int tg^2(3x)dx$	14.3	$\int cotg^2(2x)dx$

Интегриране чрез внасяне на израз зад d

15.1	$\int \dfrac{8x}{\sqrt{5 - x^2}} dx$	15.2	$\int \dfrac{2x}{\sqrt{6 - x^2}} dx$	15.3	$\int \dfrac{4x}{\sqrt{3 - x^2}} dx$
16.1	$\int \dfrac{2x}{x^2 + 1} dx$	16.2	$\int \dfrac{3x}{x^2 + 4} dx$	16.3	$\int \dfrac{4x}{x^2 + 9} dx$
17.1	$\int \dfrac{\cos(x)}{sin^2(x) - 9} dx$	17.2	$\int \dfrac{2\sin(x)}{cos^2(x) + 9} dx$	17.3	$\int \dfrac{\sin(x)}{cos^2(x) - 4} dx$
18.1	$\int e^x \sin(2e^x - 1)\, dx$	18.2	$\int e^x \sin(3e^x + 1)\, dx$	18.3	$\int e^x \sin(6e^x - 5)\, dx$
19.1	$\int 3^x(3^{x+1} - 1)^{14}\, dx$	19.2	$\int 5^x(5^{x+1} - 1)^{24}\, dx$	19.3	$\int 7^x(7^{x+1} - 1)^{17}\, dx$
20.1	$\int tg(2x)dx$	20.2	$\int cotg(2x)dx$	20.3	$\int tg(3x)dx$
21.1	$\int sin^3(2x + 3)dx$	21.2	$\int sin^5(x + 2)dx$	21.3	$\int sin^7(2x - 1)dx$
22.1	$\int cos^7(x + 3)dx$	22.2	$\int cos^3(2x + 3)dx$	22.3	$\int cos^5(3x + 1)dx$
23.1	$\int \dfrac{ln^3(x)}{x} dx$	23.2	$\int \dfrac{ln^4(x)}{x} dx$	23.3	$\int \dfrac{ln^5(x)}{x} dx$
24.1	$\int \dfrac{\sqrt[4]{ln^3(x)}}{x} dx$	24.2	$\int \dfrac{\sqrt[5]{ln^3(x)}}{x} dx$	24.3	$\int \dfrac{\sqrt[3]{ln^2(x)}}{x} dx$

Интегриране по части

25.1	$\int x.\sin(x)dx$	25.2	$\int x.\cos(x)dx$	25.3	$\int x.e^x dx$
26.1	$\int x.e^{-2x}dx$	26.2	$\int x.\sin(3 - x)dx$	26.3	$\int x.\cos(3x)dx$
27.1	$\int x^2.\cos(3x)dx$	27.2	$\int x^2.e^{-2x}dx$	27.3	$\int x^2.\sin(3 - x)dx$

28.1	$\int x.\ln(x)dx$	28.2	$\int x^2.\ln(x)dx$	28.3	$\int x^3.\ln(x)dx$
29.1	$\int x^3.\ln(x-1)dx$	29.2	$\int x.\ln(x-2)dx$	29.3	$\int x^2.\ln(x+1)dx$
30.1	$\int x^2.ln^2(x)dx$	30.2	$\int x^3.ln^2(x)dx$	30.3	$\int x.ln^2(x)dx$
31.1	$\int \ln(2x-1)dx$	31.2	$\int \ln(2-x)dx$	31.3	$\int \ln(2-3x)dx$
32.1	$\int \mathrm{arctg}(x)dx$	32.2	$\int \mathrm{arccotg}(x)dx$	32.3	$\int \arcsin(x)dx$
33.1	$\int x.\mathrm{arccotg}(x)dx$	33.2	$\int \arccos(x)dx$	33.3	$\int x.\mathrm{arctg}(x)dx$
Интегриране чрез полагане					
34.1	$\int \dfrac{2}{\sqrt{3-2x-x^2}}dx$	34.2	$\int \dfrac{1}{\sqrt{3+2x-x^2}}dx$	34.3	$\int \dfrac{5}{\sqrt{7-4x-x^2}}dx$
35.1	$\int \dfrac{1}{\sqrt{x^2+4x-5}}dx$	35.2	$\int \dfrac{2}{\sqrt{x^2+2x-8}}dx$	35.3	$\int \dfrac{1}{\sqrt{x^2+6x-10}}dx$
36.1	$\int \dfrac{7-12x}{\sqrt{4x-3}}dx$	36.2	$\int \dfrac{7-12x}{\sqrt{2x-3}}dx$	36.3	$\int \dfrac{12x-5}{\sqrt{2-3x}}dx$
37.1	$\int \dfrac{1}{\sqrt{2-x}-\sqrt[3]{2-x}}dx$	37.2	$\int \dfrac{1}{\sqrt{x-1}-\sqrt[3]{x-1}}dx$	37.3	$\int \dfrac{1}{\sqrt{x+2}-\sqrt[3]{x+2}}dx$
38.1	$\int \dfrac{1}{1+\sin(x)}dx$	38.2	$\int \dfrac{1}{1+\cos(x)}dx$	38.3	$\int \dfrac{1}{2-\sin(x)}dx$

Пожелавам ви приятно и успешно решаване на задачите!
Доц. д-р Матева
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